**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

1. 0.3875
2. 0.2676
3. 0.5
4. 0.6987

Ans :

Z= (x-u)/s.d. = (60- (45+10))/ 8 = 0.625

p = table value = 0.7324 for SM meet his commitment

p = 1- 0.7324 = 0.2676

The probability that the service manager cannot

meet his commitment is 0.2676 = 26.76%

Right option is option b

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

ANS :

Probability of clerical employees at an insurance claims processing center are older than 44 is Z score = (Value - Mean)/SD

=(44-38)/6 = 1

p value fron z table for 1 is 0.8413 mean 84% of 400

Older than 44 = (1-0.8413) = 0.16 = 16% of 400 means 64 employee from 400

For 38 = (Value - Mean)/SD

= (38-38)/6 = 0 / 6 = 0 p value fron z table for 0 is 0.5 means 50% of 400

Between 38 and 44 = p(44) - p(38) = 0.8413 – 0.5 = 0.3413 means 34 % of 400

136 employees are there in between 38 and 44 years

here employees between age 38 and 44 are 136 and older than 44 are 64 ,hence statement is **false**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans : Probability of employees at an insurance claims processing center under age of 30

Z score = (Value - Mean)/SD  = (30-38)/6 = -1.33

p value fron z table for -1.33 is 0.0918 mean 9.18% of 400 = 36 employee **TRUE**

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS :

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| Ans- As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) . | |
|  |  |
| Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ). Therefore in the question 2X1~ N(2 u,4 σ^2) and X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 ) 2X1-(X1+X2) = N( 4µ,6 σ^2) | | |  |
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1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans :

Z value is given as stats.norm.ppf(pvalue)

Z(0.99) is 2.576

Z score = (Value - Mean)/SD

value = (Z score \* SD) + - Mean = (2.576 \* 20 )+ 100

value = 151.52

Given - symmetric about the mean

100-51.5 = 48.5

48.5 100 151.5

a mean b

option D is right ANSWER

5 . Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ams : Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans : 5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans : probability of division 1 = stats**.**norm**.**cdf(0,5,3) = 0.0477903522728147

probability of division 2 = stats**.**norm**.**cdf(0,7,4) = 0.040059156863817086

Therefore division 1 has larger probability of making loss in year